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Efficient Airspace Scheduling for Military Exercises: A 3D Bin Packing Approach

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Abstract: Emerald Flag is a military exercise hosted by the 96th Test Wing at Eglin Air Force Base, FL. In its latest iteration, 10 aircraft platforms flew 26 total missions over two days. The current manual airspace allocation process is not only highly time-consuming but also exponentially growing in complexity as more participants join. Our project aims to minimize time dedicated to scheduling by developing a 3D Bin Packing Problem (3D-BPP) model limited by unique airspace assignment constraints. Our model's objective is to maximize the number of missions per day whilst adhering to safety requirements. Utilizing scheduling requests crafted by the 96th, we find the model generates a feasible schedule in 80 fewer man-hours than current processes and increases daily mission capacity by 18 missions while maintaining operational safety. In conclusion, the 3D-BPP model is a flexible, efficient solution for military test range scheduling that is easily adaptable for similar exercises.

Keywords: 3D Bin Packing Problem, Airspace Allocation, Scheduling Optimization

1. Introduction

Emerald Flag is an exercise held triannually by the 96th Test Wing at Eglin Air Force Base, Florida. Emerald Flag provides both civilian and military participants a platform to test new and existing aircraft, weapons, and sensor equipment. The exercise takes place over a week, employing two days specifically for execution. Currently, Emerald Flag's execution schedule is created by a planning team that assigns participants to specific airspace and test resources by hand using a whiteboard. This planning process includes 3 separate multi-day meetings which ultimately results in an airspace plan. During the execution of the exercise, planners track airspace assignments on a physical map using color-coded pins. After three years of execution, Emerald Flag has seen a growth from 41 missions flown over the three exercises held in FY2021 to 56 total missions in FY2023. While it is currently possible to manually draft a schedule that satisfies all testing requirements, this process is challenging and time-consuming. As more aerial participants join the exercise, the airspace gets busier, making the process increasingly intricate. Each test mission in the schedule requires careful consideration and conversation to ensure safe operation for all parties. Variables include the multiple requirements and restrictions relating to a mission type, aircraft location relative to the shoreline, and space from other aircraft, all of which force schedulers to continually reevaluate if each airframe's requirements are met. This manual scheduling process results in possible inefficiencies in schedule creation speed and airspace allocation.

1.1 Problem Statement

The 96th Test Wing needs to allocate a portion of airspace in the test range to each exercise participant. However, doing so by hand is challenging, time-consuming, and can lead to inefficient schedules. Possible inefficiencies arise in the forms of suboptimal airspace allocation and distant aircraft placement, leading to wasted fuel and travel time, among others.

1.2 Related Work

The assignment problem can be modeled similar to a 3D-BPP except instead of multiple bins to pack we have a single bin, representing the entire airspace. Christensen, Khan, Pokutta, and Tetali (2017) provide an overview of the research and methods related to bin packing problems. To switch from a manual grid assignment system to the 3D bin packing system, its variations must be understood. Martello, Pinsger, and Vigo (1998) explored the different objectives a 3D-BPP can solve. Two types were the Knapsack Loading and Container Packing problem. The Knapsack Loading problem involves selecting which subset of a list of items can fit into a single bin to maximize the collective profit. While our problem consists of a single bin (i.e., airspace), we must assign all the items (i.e. aircraft) in the bin such that participants are as close to the shoreline as possible without violating safety constraints.

The Container Packing problem is a geometric assignment problem where the objective is to fit a list of multiple 3D items into the minimum number of bins at a set volume. George (1992) provided the mathematical foundation needed when fitting items that consist of different 3D shapes and introduced the concept of layering the 3D shapes within a singular bin. When allocating airspace, each airframe has unique altitude and safety requirements while simultaneously having the ability to be staggered in all three dimensions. Lim, Rodriguez and Yang (2005) made advancements with a number of feasible solutions for bin packing by including item orientation. The algorithm developed by the team allows for the dimensions of the x,y,z coordinates to be interchanged so the boxes have more flexibility in their positioning. As featured in the paper, the ability to orient the aircrafts' volume requirements will provide a more accurate picture of how they may be placed, however our problem will only require x and y rotation because z (i.e. altitude) will be treated as a constraint.

1.3 Organization

Our paper begins with the methodology section, where the applications of the data obtained from the 96th Test Squadron and our methodological approach are addressed. Following the methodology, the paper transitions into the results and analysis section, presenting the outcomes and interpretation of our model. The final section encompasses conclusions, recommendations, and avenues for further research.

2. Methodology

2.1 Data Collection and Analysis

Communication cards provided by the organizers of Emerald Flag were our primary source of data. CCs provide a summary of each participant's flight information, such as assigned airspace, altitudes, and radio frequencies. We received 11 out of 18 CCs between exercises 21-1 and 23-3. The seven missing cards either contained un-releasable information or did not include a meaningful number of participants. Each was analyzed for the number of participants and missions executed as shown in Figure 1 below.

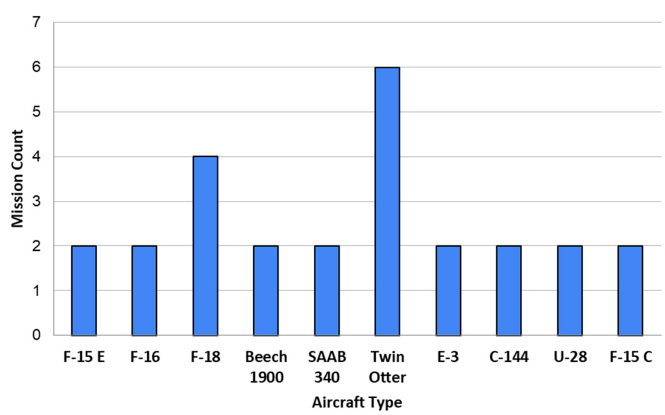

Figure 1: 2023-2: Missions Flown vs. Aircraft Type

A noticeable increase in both metrics is evident between each exercise, and organizers have indicated that overbooking will eventually become a problem. Separately, a file of the overall airspace coordinates provided the dimensions of the exercises' airspace bin. A brief visual analysis of the grid space map was performed to search for assumptions and constraints that manual schedulers currently follow that were later accepted or rejected in discussions with the organizers.

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Additionally, the team attended one of three planning conferences to learn about the scheduling process and the exercise lead priorities to better inform model design. While there, Emerald Flag organizers also provided artificial CCs to test the robustness of our model. These CCs cover a variety of test cases, ranging from a standard number to an abnormally high number of participants.

2.2 Model Methodology

Our model falls under the umbrella of the 3D-BPP; however, additional complexities are considered to accommodate the safety requirements of airspace allocation. Plainly put, the 3D-BPP solves the issue of efficiently packing boxes inside bins to minimize the number of bins required. Instead of multiple bins, our model packs boxes (i.e. aircraft flight zones) into a singular bin (i.e. exercise airspace) that cannot be freely expanded. For the purposes of Emerald Flag, minimizing wasted space inside the bin is correlated with maximizing the number of missions flown. The model addresses two key issues: given a list of participants, is there a feasible allocation of airspace and where will the participants be placed? The primary measure of performance (MOP) will be the daily flying mission capacity. The distance between each participant and shoreline, the speed of the allocation process, and the reduction in time required to produce the execution schedule compared to current methods will be secondary MOPs. Ultimately, the aim is to reduce time spent on scheduling while producing a feasible airspace allocation that maximizes these MOPs.

In the model, each object will be depicted by a rectangular prism, representing a block of airspace. The dimensions of each block are measured in tens of thousands of feet. Aspects of both the aircraft and the participant's requirements will affect the size of the block. Model constraints include distance from the south coastline, minimum and maximum altitude, and international water requirements. Mathematically, the entire model is represented by the following variables, objective function, and constraints:

Parameters:

L, W, H : length, width, and height dimensions of airspace
n : number of total boxes, where boxes represent an individual mission
B: set of all boxes $\{0, n-1\}$
l_i, w_i, h_i : length, width, and height dimensions of box *i*
c_i : min operating distance from coastline, represented by the x-axis
a_i : min flying altitude of box *i*
b_i : max flying altitude of box *i*
u_i : binary indicator for if box *i* represents a no fly zone (1 if NFZ)
d_i, e_i, f_i : predetermined x,y,z coordinates for location of NFZ *i*

Decision Variables:

x_i : x-location of box *i*'s bottom northwest corner	$x_i \in \mathbb{R}$ s.t. $0 \leq x_i + l_i \leq L$
y_i: y-location of box *i*'s bottom northwest corner	$y_i \in \mathbb{R}$ s.t $0 \leq y_i + w_i \leq W$
z_i: z-location of box *i*'s bottom northwest corner	$z_i \in \mathbb{R}$ s.t. $0 \leq z_i + h_i \leq H$
$o_{x,ij}$: overlap indicator between boxes *i* and *j* on x-axis	$o_{x,ij} \in \{0,1\}$ where $i, j \in B$
$o_{y,ij}$: overlap indicator between boxes *i* and *j* on y-axis	$o_{y,ij} \in \{0,1\}$ where $i, j \in B$
$o_{z,ij}$: overlap indicator between boxes *i* and *j* on z-axis	$o_{z,ij} \in \{0,1\}$ where $i, j \in B$
r_i: binary indicator for if box *i* is rotated (0 not rotated)	$r_i \in \{0,1\}$ where $i \in B$

Mathematical Formulation:

$$\text{Min} \sum_{i=0}^{B} \left(y_i + \frac{(1-r_i)\cdot w_i + r_i \cdot l_i}{2} + z_i + \frac{h_i}{2}\right) \tag{1}$$

Subject to:

$$0 \leq x_i + (1-r_i)\cdot l_i + r_i \cdot w_i \leq L \qquad \forall\, i \in B \tag{2}$$
$$0 \leq y_i + (1-r_i)\cdot w_i + r_i \cdot l_i \leq W \qquad \forall\, i \in B \tag{3}$$
$$0 \leq z_i + h_i \leq H \qquad \forall\, i \in B \tag{4}$$
$$x_i \geq x_j + (1-r_j)\cdot l_j + r_j \cdot w_j - L \cdot o_{x,ij} \qquad \forall\, i,j \in B, i \neq j \tag{5}$$
$$y_i \geq y_j + (1-r_j)\cdot w_j + r_j \cdot l_j - W \cdot o_{y,ij} \qquad \forall\, i,j \in B, i \neq j \tag{6}$$
$$z_i \geq z_j + h_j - H \cdot o_{z,ij} \qquad \forall\, i,j \in B, i \neq j \tag{7}$$

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$$o_{x,ij} + o_{y,ij} + o_{z,ij} \leq 2 \qquad \forall\, i,j \in B, i \neq j \qquad (8)$$
$$z_i \geq a_i \qquad \forall\, i \in B \qquad (9)$$
$$z_i + h_i \leq b_i \qquad \forall\, i \in B \qquad (10)$$
$$y_i \geq c_i \qquad \forall\, i \in B \qquad (11)$$
$$x_i = d_i, y_i = e_i, z_i = f_i \qquad \forall\, i \in B \text{ if } u_i = 1 \qquad (12)$$

The objective function (1) seeks to minimize the distance between the geometric center of each participant's airspace zone and the southern coastline (i.e. x-axis). The y variable measures box i's distance from the coastline. The model is then bounded by the following constraints. Equations (2-4) ensure the entire volume of box i remains within the container dimensions. (5-7) ensure box i is placed beyond the end of box j in all three dimensions. (8) works in conjunction with the previous three constraints, preventing boxes from overlapping in each dimension. Constraint (9) ensures box i is located above its altitude floor, while (10) ensures the height of box i does not exceed its altitude ceiling. Constraint (11) ensures that box i is placed beyond its minimum operating distance from the coastline. Finally, constraint (12) ensures NFZ boxes are set at their predetermined locations.

To translate the mathematical formulation into a computational framework, we leveraged Python and the Gurobi Optimizer Engine. Data inputs in Excel undergo preprocessing before Gurobi executes optimization and generates an output. This model is built on the following assumptions with regard to the missions and their respective aircraft. To mitigate real-world complexity, factors including weather, maintenance issues, and airspace interference are presumed negligible. Additionally, aircraft will not leave the airspace they are assigned except for entry and exit at the start and end of the exercise. Each rectangular airspace will not intersect with any other aircraft's airspace. Aircraft will take off with enough fuel required for the full test window. All participants listed will be assigned airspace unless it is infeasible, in which case the user will decide which participant to remove from the list. Finally, to better represent the true available airspace, NFZs were introduced through the use of *ghost boxes*, or airspace blocks associated with hard constraints that force them into their respective locations. Thus, this helps prevent aircraft placement within NFZs above public lands.

Following completion and implementation of the initial algorithm, we tested the model on small samples of up to six aircraft to guarantee functionality as shown in Table 2. With each iteration of the model, additional constraints and test cases were performed to ensure the model properly executed our mathematical formulation.

Table 2. 3D-BPP Model Input

Callsign	Plane	Length (x)	Width (y)	Height (z)	Min Alt.	Alt. Cap	Priority	Int. Waters	NFZ
WHALE 01	P-3	40	40	20	0	20	1	0	0
ZEUS 30	MQ-9	15	10	5	40	50	2	0	0
DEER 29	B-52	20	20	10	0	50	1	0	0
WOLF 10	C-208	10	15	5	0	12	2	0	0
FISH01*	F-15C	20	15	20	0	50	1	1	0
MIKE 02	EC-130	20	20	20	15	35	2	0	0
NFZ 1	NFZ	30	20	30	0	30	0	0	1

3. Results and Analysis

Given a list of participants, our model successfully addresses whether it is feasible to assign all participants and if so outputs each participant's designated airspace. Table 3 displays the model's allocation output of Table 2's participant roster. Each position output is the bottom northwest corner of each aircraft's allocated flight zone and rotation addresses the boxes' dimensions in the x and y-axis. Figure 2 provides our 3D visualization of the model's feasible airspace allocation. Axes are measured in tens of thousands of feet.

Table 3. 3D-BPP Model Output

Callsign	Plane	Position	Rotation
WHALE 01	P-3	[20,20,0]	0
ZEUS 30	MQ-9	[45,0,40]	0
DEER 29	B-52	[30,0,0]	0
WOLF 10	C-208	[45,0,0]	1
FISH01*	F-15C	[30,0,0]	1
MIKE 02	EC-130	[0,20,15]	0
NFZ 1	NFZ	[0,0,0]	0

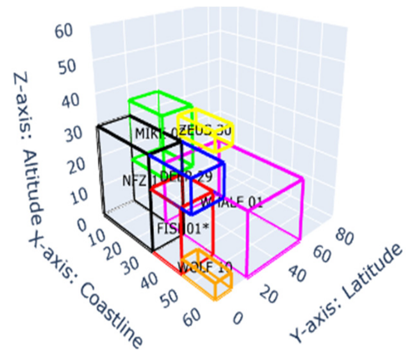

Figure 2. 3D Visualization of Airspace Allocation

The model also provides 2D visualizations of the output, divided into three altitude ranges. They are 0-10, 11-20, and 21-60 thousand feet. These simply provide users an additional perspective of the airspace allocation.

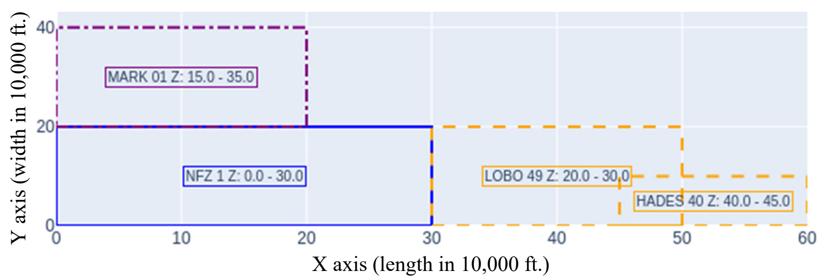

Figure 3. 2D Visualization of Airspace Allocation (Altitude 21,000-60,000 ft.)

The next step was evaluating our model's scheduling efficiency by inputting various larger test cases including historical data from Emerald Flag exercises and artificially generated scenarios provided by organizers. Our sensitivity analysis began with estimating the average distance participants are from the coastline or the international water threshold. To conduct the average distance participants are from their preferred origin, airspace volumes were categorized into four ranges. We compiled the allocation results from seven test cases including a total of 80 aircraft, shown in Table 4 below.

Table 4. Average Distance vs. Aircraft Volume

Airspace Volume Range (100,000,000,000 ft^3)	Avg. Distance from Coastline (ft)
(600 - 2,000)	2500
(2,001 - 5,000)	1333
(5,001 - 11,000)	3333
(11,001 - 60,000)	16667

This analysis validates our model's efficiency in positioning aircraft close to their preferred locations. However, it also reveals distinctive airspace allocation patterns. Our model demonstrates a pattern of clustering smaller volume boxes closer to the coastline, while larger boxes are consistently positioned farther away. We attribute this pattern to the formulation of our objective function. By doing so, it ensures that aircraft of varying sizes and operational needs are accommodated effectively, while also maintaining a fluid and orderly airspace structure.

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One crucial aspect of the evaluation was measuring time required to generate outputs for larger scenarios. Smaller scenarios up to 6 aircraft averaged 0.8 seconds. However, larger efficiency gains come into play with larger samples. Even with the increased complexity, the model produced an output within an average of 5.5 seconds if feasible. The impact is substantial, reducing time spent on draft airspace allocations by an estimated 80 work-hours.

We also explored an option where the size of airspace boxes was adjusted. This analysis aimed to identify the average increase that could be accommodated during the exercise without adjusting the outputted airspace design. We initially established a baseline by analyzing the current allocation patterns and safety margins. Our findings indicated that, on average, airspace boxes could be expanded by up to 19% without impinging on adjacent airspace allocations. This expansion was more feasible in areas further from the coastline, where the density of aircraft was lower and the risk of conflict less acute. In contrast, near the coastline, where airspace is at a premium, the potential expansion was limited to around 4%. A baseline margin of 15% also indicates a built-in amount of safety margin between airspace allocations.

To quantify this, we ran simulations with artificially increased box sizes in varying increments, from 5% to 20%, while monitoring for loss of model feasibility. The results showed that increasing box sizes beyond 17% led to a significant increase in infeasibility across various scenarios provided by organizers. Therefore, we concluded that a safe and efficient expansion could be achieved within the 4% to 16% range, depending on existing airspace utilization inside the scenario. This adjustment in box sizes could allow for a more flexible allocation of airspace, especially during high-density flight operations.

4. Conclusions, Recommendations, and Future Research

Our study demonstrates the successful implementation of a 3D-BPP model to optimize airspace allocation for the Emerald Flag military exercise. By minimizing wasted airspace and efficiently positioning aircraft, our model significantly reduces the time and effort required for scheduling, yielding a feasible schedule in 80 fewer work-hours compared to current manual processes. Such a reduction has ripple effects across the entire process, allowing organizers more time to focus on fine-tuning allocations and analysis of results. Furthermore, the model enables the expansion of participant capacity while assuring safety considerations during execution. Adoption of a 3D-BPP model will allow Emerald Flag and other department and joint exercises to scale in capacity while maintaining safety considerations. By optimizing distance from a priority axis, exercise planners will be able to ensure the maximum amount of time-on-station during execution. This will reduce overall event costs and increase the amount of data that can be collected.

However, it must be remembered that this model does not solve the problem of hour-by-hour scheduling, communication frequency assignment, and other tasks required for Emerald Flag. Rather, it only serves as a tool for the participants' airspace assignments. Limitations exist in the forms previously listed, as well as inabilities to recognize human error and inconsistencies in data inputs that could compromise the integrity of the model and its outputs.

To further refine the model, we suggest enhancing the representation of coastal zones to better account for their complex geometry. Additionally, implementing a more user-friendly interface within the model could improve data input reliability and user interaction, thereby increasing overall efficiency and accuracy.

Future research should focus on expanding the model's functionality, including the integration of an additional bin to represent the second execution day during the exercise. Introducing built-in participant removal functionality could offer users greater flexibility in adjusting schedules without compromising the overall feasibility of the allocation. Implementing a function to lock specific boxes and rerun the model without altering locked selections could streamline the scheduling process by allowing organizers to prioritize certain participants while adjusting others. To better visualize airspace allocations, overlaying the 2D and 3D visualizations onto actual geographic maps could provide a more intuitive and practical tool for exercise planning. Together, these enhancements could improve the model's adaptability and effectiveness in addressing the nuanced demands of military exercise planning.

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